MERELY CONFUSED SUPPOSITION: A THEORETICAL ADVANCE OR A MERE CONFUSION?
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# MERELY CONFUSED SUPPOSITION

## A THEORETICAL ADVANCE OR A MERE CONFUSION?

### Introduction

In this article we will discuss the notion of merely confused supposition as it arose in the mediaeval theory of *suppositio personalis*. The context of the analysis is our formalization of William of Ockham's theory of supposition sketched elsewhere.[1] The present paper is however, self-contained, although we assume a basic acquaintance with supposition theory. The detailed aims of the paper are: i) to look at the tasks that supposition theory took on itself and to use our formalization to relate them to more modern ideas; ii) to explain the notion of merely confused supposition and to defend it against certain criticisms; and iii) to discuss two issues closely related to the idea of merely confused supposition which we would not broach in a shorter article: the mode of supposition of terms in intensional contexts, and the possible existence of a fourth mode, often called *suppositio copulatim*.

Our account is essentially a theoretical one which attempts to explain and account for a number of features of mediaeval supposition theory and, as such, it is ultimately to be tested against source material. Hence, although much of the evidence we shall offer comes from Ockham's writings, our conclusions apply to the mediaeval theory of supposition, in terms of descent, as it is found in a number of authors. Our earlier article drew on three works of Ockham's: *Summa Totius Logicae* (henceforth *STL*), *Elementarium Logicae* (*EL*), and *Tractatus Logicae Minor* (*TLM*).[2] Since then we have be-

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[1] G. Priest & S. Read, "The Formalization of Ockham's Theory of Supposition," *Mind*, 86 (1977), 109–13.

[2] STL: G. de Ockham, *Summa Logicae*, eds. P. Boehner, G. Gál & S. Brown

come aware of Gál's persuasive doubts concerning the authenticity of the last two works. His conclusion is that they cannot be taken as genuine texts of Ockham's until further research has been carried out.[3] We will find it possible here to draw most often on *STL*; however, the reader will note that our findings are consonant with both *TLM* and *EL*, and on occasions we will look there for support, and also to writings by later authors working in the same tradition.

## 1. The Theory of Supposition and its Function

The current notion of reference has a narrow use and a wide one. In the narrow sense of 'reference,' it is only singular terms that can have reference, for example 'Russell,' 'that man,' 'the winner of the race.' In the wider sense of 'reference,' also called 'extension,' one can attribute reference not only to singular terms but also to general terms such as 'man,' 'men holding babies,' and even to quantified terms such as 'all women,' and 'some woman holding a baby.' These terms relate to an object or set of objects whose properties are important in determining the truth or falsity of sentences in which the terms occur. This object or set of objects is the reference of the term in question. The mediaeval theory of personal supposition is a theory of reference in the wider sense of the term.

A singular term was said to have *discrete supposition* for an object. A general or quantified term was said to have *common supposition* for the class of objects involved. (A general term such as 'man' supposited for every member of the class of men; so 'some men' supposits not for some men but for all men.) Common supposition was, in turn, of different kinds. We will turn to this division in § 4. What we need to note here is that only in the context of a sentence did a categorematic term have supposition. (Recall Frege's doctrine that only in the context of a proposition does a word have *Bedeutung*.) Moreover, the supposition of a term in a sentence is a measure of the way in which a term stands for certain objects, namely those in the extension of the term.

(*Opera Philosophica* I: St. Bonaventure, 1974). *EL*: "The *Elementarium Logicae* of Ockham," ed. E. Buytaert, *Franciscan Studies*, 25 (1965), 151–276 and 26 (1966), 66–173. *TLM*: "*Tractatus Logicae Minor* of Ockham," ed. E. Buytaert, *Franciscan Studies*, 24 (1964), 34–100.

[3] *STL*, pp. 62*–66*.

So much for what the theory of supposition was. We must now turn to its most important use. The central concern of logic is the notion of validity. Logicians also concern themselves with a syntactic notion of derivability, but it is ultimately responsible to the semantic notion of validity. How to characterise validity is still a problem. A common way of doing it both in the twentieth century and in the thirteenth is in terms of truth preservation. Certainly this was the dominant approach in medieval logic.[4] Thus to give an account of validity (and invalidity) the medievals had to give an account of the truth conditions of the sentences which were candidates for implying and being implied. This was the main use to which the theory of supposition was put. Just as moderns give the truth conditions in terms of (world-dependent) reference and extension, so the mediaevals gave truth conditions in terms of supposition. The point is an important one and bears repeating.

In order to give a justification for claims about the validity or invalidity of certain inferences, in syllogistic or eventually more generally, mediaeval authors needed a theoretical apparatus within which to give the truth conditions of sentences: supposition theory. A simple example of the use of the theory in giving truth conditions—we will meet more complex cases later—was Ockham's infamous doctrine that a singular sentence is true if the subject and predicate "supposit for the same thing." Now one of the most important applications of truth conditional semantics is in pinpointing invalidity. In modern logic we have the notion of a counter-model. The mediaevals used their supposition theory for exactly the same purpose. This explains the central role of supposition theory in the discussion of *sophismata* and the doctrines concerning fallacy.

One of the most important logical events of the last century was Frege's introduction of the structural analysis of quantified terms: the representation of the quantifier as an operator on a propositional function or predicate. In this way Frege was able to solve a number of problems concerning multiple generality. The mediaevals

[4] For example, Pseudo-Scotus defined validity in this way: "A consequence is a hypothetical sentence composed of an antecedent and a consequent, connected by a conditional or inferential connective, which means that it is impossible that, when the antecedent and the consequent are formed simultaneously, the antecedent is true and the consequent false. And then, if it is as this connective says, the consequence is valid; and if not, then the consequence is invalid." *Ioannis Duns Scoti Opera Omnia*, ed. L. Wadding (Paris, 1891), II, 104–5.

did not take this step. But with the notion of *descensus* whereby the supposition of a quantified term in a sentence is analysed in terms of conjunctions or disjunctions of terms with discrete supposition, they made a similar structural move. (Such Boolean combinations of singular sentences were later called *descendentes*—"descended forms," and in fact are conjunctive and disjunctive normal forms of a certain kind.) The connection between universal quantification and conjunction, and existential quantification and disjunction has often been remarked. If we have a name for every object in the domain and allow conjunctions and disjunctions of the same cardinality as the domain then a universally quantified sentence is materially equivalent to the conjunction of its instances and dually for an existentially quantified sentence. It is perhaps not surprising therefore that the mediaevals were able to solve the problems of multiple generality, which Frege solved with the quantifier, with the doctrines of descent connected with supposition theory. For example, they could account for the validity of the inference from 'Some man was inside the whole day' to 'The whole day some man was inside' and the invalidity of its converse. They could also recognize and exhibit the ambiguity of a sentence such as 'Some man loves every woman.' The Fregean move is to account for the ambiguity of this in terms of the scope of the quantifiers. The mediaevals accounted for the ambiguity in terms of modes of supposition which were ultimately cashed out in terms of the relative scopes of conjunctions and disjunctions. The mediaevals could, with their account of supposition, begin to give a semantics of inference.

## 2. The *Descensus* as requiring Equivalence

We have described supposition theory as essentially a theory of reference in terms of which an account of truth conditions could be given, and have claimed in particular that the descended form analysing a sentence (where one exists) is to be understood as giving the truth conditions of the original sentence in terms of sentences with discrete supposition. If this interpretation is correct then the descended form must be materially equivalent to the original sentence.

That the descended forms are materially equivalent to the forms before descent is part of the folklore, normally unargued for, about supposition theory. However, the idea has recently been challenged in an interesting article by Corcoran and Swiniarski.[5] They argue that a more coherent understanding can be gained of Ockham's account of the modes of supposition when it is realised that descended forms were not intended to be equivalent to the original sentences. In particular, they argue that a mode of supposition was characterized by Ockham not only by what kinds of descent are possible but also by the possibilities for ascent, that is, inference from one of the singulars in the descended form.

There are several comments to be made on this claim. First, if they are correct then the specific details of the formalization we shall present are certainly wrong. The general tenor of our formalization is nonetheless correct and it can easily be modified to fit their account. But we feel that their account is importantly incorrect.

Looking at the theory of supposition from its inception in the early thirteenth century to its disappearance in the seventeenth, we can trace a certain development, especially in the notion of descent. Many of the earliest thinkers such as Peter of Spain and Lambert of Auxerre[6] made no use of descent in the definitions of modes of supposition. If they mentioned descent at all it was as a consequence of the definitions of these modes. Over the next hundred years thinkkers such as William of Sherwood, Roger Bacon and Walter Burleigh[7] did use descent to define at least some of the modes. However, descent was not made to a disjunction or conjunction of singular sentences, but to a single sentence. In Sherwood and Bacon, descent was made to any singular—that is, it was necessarily conjunctive. In later writers, the descent was first said to be performed disjunctively (*disiunctive*) or conjunctively (*copulative*), and then later to be performed by a disjunctive or conjunctive sentence (*per disiunctivam vel copulativam*). Sometimes ascent conditions were also involved in the definitions of the modes of supposition, but again it was always the possibility of ascent from one singular that was in question. By

[5] J. Corcoran & J. Swiniarski, "Logical Structures of Ockham's Theory of Supposition," *Franciscan Studies*, 38 (1978), 161–83.

[6] Peter of Spain, *Tractatus*, ed. L. M. de Rijk (Assen, 1972); Lambert of Auxerre, *Logica*, ed. F. Alessio (Firenze, 1971).

[7] William of Sherwood, *Introductiones in Logicam*, ed. M. Grabmann, *Sitzungsberichte der Bayerischen Akademie des Wissenschaften*, Philosophische–Historische Klasse, Jahrgang 1937, Heft 10; Roger Bacon, *Sumule Dialectices*, ed. R. Steele (Oxford, 1940); Walter Burleigh, *De Puritate Artis Logicae Tractatus Longior*, ed. P. Boehner (St. Bonaventure, 1955).

about the middle of the fourteenth century the theory of supposition had settled down into more or less its finished form, as we find it in Albert of Saxony and fifty years later in Paul of Venice's major compilation.[8] The modes of supposition are defined in terms of descent conditions and descent is to an equivalent single Boolean combination of singular sentences. If ascent is mentioned at all, it is ascent from the whole descended form, not from just one singular.

The most important step in the above process is the transition from descending *disiunctive* (or *copulative*) to a single singular to descending (*simpliciter*) to a disjunction (or conjunction) of singulars. Once this had been made it became clear that the descended forms are as a matter of fact materially equivalent to the undescended ones. They could therefore be taken as giving the truth conditions of the original sentences in a particularly simple way.[9]

Ockham's place in this history is an interesting one. He is in the final group as one of those who recognised descent to a disjunctive or conjunctive sentence. (Indeed, as we shall see in the next section, with some of his contemporaries he even extended this kind of analysis to ensure that such a descent was possible in all cases.) However, he is one of the earliest persons in this group and it is clear from the way that he expresses himself that he had not completely extracted himself from the earlier way of looking at things.[10] Indeed, we would claim that Ockham is essentially responsible for the final form of supposition theory, though in his work there are still to be found elements of the older approach. Such is to be expected in any new scientific development. New ways of looking at things do not emerge complete and in one go. Characteristically, the thinker who first propounds the new view is still fighting to disentangle himself from

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[8] Albert of Saxony, *Perutilis Logica* (Venice 1522; reprint Hildesheim, 1974); Paul of Venice, *Logica Magna*, tract 2, ed. (in part) A. R. Perreiah (St. Bonaventure, 1971).

[9] We can see this transition as similar to that which gave rise to the notion of objective being. In the theory of universals, '*obiective*' occurred first as an adverb, but was later reified into an adjective in the phrase '*esse obiectivum*.' (See S. Read, "The Objective Being of Ockham's *Ficta*," *Philosophical Quarterly*, 27 [1977]). In supposition theory, '*disiunctive*' became '*disiunctivum*.'

[10] Others writing at the same time in the transitional way are John Buridan, *Tractatus de Suppositionibus*, ed. M. E. Reina, *Rivista critica di storia della filosofia*, 12 (1957); Pseudo-Campsall, *Logica*, ed. E. A. Synan, in *Nine Medieval Thinkers*, ed. J. R. O'Donnell (Toronto, 1955).

the old and his work therefore bears marks of it. Thus Galileo, the originator of mechanistic dynamics, is still found using some concepts from Aristotelian dynamics, and there is more than a trace of Lamarckianism in Darwin's *Origin of Species*. So it is with Ockham: aspects of the older problematic are to be found in his writings (in particular references to ascent from one singular) and it was left to later writers such as Paul of Venice to straighten things out.

It is interesting to trace these aspects of earlier views in Ockham's writing. In *STL* I, 70, Ockham gives his definition of the modes of personal supposition; first for determinate supposition. He starts with what appears to be the definition (ll. 19–20): "Supposition is determinate when it is permissible to descend by some disjunctive [sentence] to singulars." Note the mixture of old and new: he uses the idea of a full disjunctive sentence, but that descent is to singulars—that is, singular sentences, not to the disjunctive sentence, but by it—'*per disiunctivam.*' That is all he gives in the way of a definition; he immediately gives an example. But a few lines later he remarks on why this mode is called determinate: because one "determinate singular [sentence] alone, without the truth of any other singular, suffices for the truth of a sentence [exhibiting determinate supposition in one term]." Another example is given, and he then offers "a rule": "whenever it is permissible to descend under a general term to singulars by a disjunctive sentence, and to infer the [original] sentence from any singular [sentence], then that term has determinate personal supposition." The actual definition here given is of course, insufficient; for it fails to distinguish determinate supposition from confused and distributive. The supplement needed is the ascent condition expressed in the "rule," or, amounting to the same thing, the explanation of why the mode is called determinate, or again, the understanding that descent is to an equivalent. Thus Ockham applies the tools of the older problematic to obtain the result of equivalence between descended and undescended forms.

Ockham's definition of confused and distributive supposition is as follows: "Confused and distributive supposition occurs when... it is permissible in some way to descend by a conjunctive sentence and impossible to infer the original sentence from any of the elements of the conjunction." The conjunctive descent condition is the main condition here. What is the ascent condition doing? In effect it just ensures that the term cannot supposit both determinately and confused and distributively. For if a conjunctive descent is possible then a disjunctive descent is possible *a fortiori*, but the ascent condition rules out the equivalence of the disjunctive descent. Ockham might just as well have said, as others were to, that in confused and distributive supposition a sentence is equivalent to its conjunctive descended form.[11]

Let us now turn to Ockham's account in the same chapter of *STL* of merely confused supposition. It is specified as follows: "Personal supposition is merely confused when a general term supposits personally and it is not permissible to descend to singulars by a disjunctive [sentence]... but by a sentence with a disjunct term, and it is permissible to infer it from any singular." We are given, that is, the joint sufficiency of three conditions:

1) it is not permissible to descend by a disjunctive sentence;
2) it is permissible to descend by a sentence with a disjunct term;
3) it is permissible to infer the original sentence from any singular sentence disjunct.

Note also that 1) entails

4) it is not permissible to descend by a conjunctive sentence.

Now, strictly speaking according to the definition given, the descended form need not be equivalent to the undescended one. However, not too much should be made of this for in practise Ockham works with a different account of merely confused supposition. In *STL* II, 17, Ockham shows that certain terms in various types of sentence have merely confused supposition. He seems, quite naturally, to start by repeating the criterion for the mode. He writes: "Supposition is merely confused when it is not permissible to descend to inferiors either by a disjunctive or by a conjunctive sentence" (ll. 206–8). He repeats the same procedure in *STL* II, 18: ll.31–2 and in *STL* II, 19: ll.79–80. The criterion he works with is just 1) and 4). 2) holds vacuously (we shall see) and 3) is ditched. Again, in *STL* III–3, 10, Ockham writes: "And from this it is clear that the predi-

[11] Paul of Venice, op. cit. (see note 8), p. 94, for example, wrote: "Mobile confused and distributive supposition is the signification of a general term under which it is permissible to make a descent to all its singulars conjunctively with an appropriate middle and conversely with the same middle. For from 'This animal runs and this animal runs and so on for singulars, and these are all the animals' it follows that every animal runs. And the converse inference is valid with the same minor, which is called a singular *constantia*, as here: every animal runs and these are all the animals; therefore this animal runs and so on for singulars."

cate in such a particular necessitive has merely confused supposition, namely because it is not permissible to descend to inferiors either by a disjunctive or by a conjunctive sentence" (ll.38–41). The situation then is this: descent to a sentence with a disjunct term yields an equivalent and can always be performed. Conditions 1) and 4) are Ockham's working criterion of merely confused supposition since these rule out the other two modes, and condition 3), the ascent condition, is a vestige of Ockham's intellectual heritage that he has not yet outgrown.

In *EL* the ascent conditions appear only to distinguish confused from determinate supposition. The definitions of the modes read:

> "Determinate supposition is so called... because it is necessary for the truth of a sentence in which a term supposits in this mode that it be true for some one determinate, and this suffices... So when there is determinate supposition, it is permissible to descend to singulars... by a disjunctive sentence... Supposition is confused and distributive when it is permissible to descend from a general term to inferiors, if it has many, conjunctively, that is, by a conjunctive sentence... Supposition is merely confused when neither by a conjunctive sentence nor by a disjunctive is it permissible to descend to inferiors... Nevertheless, it is permissible to descend to a disjunct predicate" (pp. 209–10).

In the Renaissance period ascent and descent became a topic in themselves, and a number of authors used them as necessary and sufficient conditions for the modes of supposition. So used they ensure that the modes are given in terms of descent to an equivalent. Thus in Jean de Celaya we find: "Determinate supposition is a term suppositing [in such a way] that for that term it is permissible to make descent and ascent disjunctively"; confused and distributive supposition requires the permissibility of descent and ascent conjunctively, and so on.[12] We can sum up the discussion as follows. In the old supposition theory, descent and ascent were relations between a sentence and one of its singulars, the relations being of various kinds. In the new theory, descent and ascent were relations between a sentence and its whole descended form, the pair of relations together constituting a material equivalence.

Ockham's theory does not state explicitly that the result of a descent in the definitions of the modes is an equivalent. However,

[12] Jean de Celaya, *Magnae Suppositiones* (Paris, 1526), sign. b3.

what the above analysis shows is that the mature supposition theory is a theory of equivalences[13] and moreover that that theory is essentially to be found in Ockham, albeit in a form not clearly disentangled from the older way of approaching supposition theory. Corcoran and Swiniarski's account of Ockham's theory must be admitted to fit the text of *STL* I, 70 neatly. However, it fits the text of other parts of *STL* less well, EL not at all, and fails to put the development of the theory in its historical context. We consider Corcoran and Swiniarski's theory to be one designed to "save the phenomena" of the text of *STL* I, 70 alone. Given any data it is possible to construct a theory which the data fits exactly—even the data due to experimental error! However, the point of a theory is not just to fit the data but to explain it. This our theory does. It places Ockham's account at the crucial stage of the development of mature supposition theory, itself a theory with a clear rationale, namely to give the truth conditions of all sentences in terms of their fully descended forms.

## 3. Merely Confused Supposition and Nominalism

We must now discuss two important aspects of the notion of merely confused supposition. Its first importance is, in modern terms, the reduction of ontological commitment. The theory of supposition was, we have argued, a theory of truth conditions, and the ontological commitment of a sentence was thought to consist of those entities for which terms had to supposit in order adequately to state the truth conditions of the sentence. Thirteenth century logicians could see how to make the descent to singulars in a number of cases (those which answered to determinate and distributive supposition) and hence could give truth conditions in those cases in terms of the descended forms. But in other cases, the predicate of A–sentences, for example, they could not take this line. Instead they said that such a term had simple supposition, that is, it stood for the universal or nature expressed by the general term. The truth conditions of an A–sentence were then given in terms of the objects for which the subject supposited, "partaking" in the universal for which the pre-

[13] Any doubts of Russell's sort that one needs a clause saying that these are all the instances were shared by some of the mediaevals; their solution was the *constantia* mentioned in the earlier quotation from Paul of Venice (see note 11).

dicate supposited. Thus thirteenth century realist logicians such as Peter of Spain were committed to the existence of such abstract entities. This was unappealing to the nominalist thought of Ockham and others, and they hit upon a simple idea to give the truth conditions of such sentences without needing to refer to universals. They did this by using the notion of a disjunction of names.

Before we elaborate on this point it is important to note that Ockham did not dispose of the notion of simple supposition altogether. According to him, 'man' in 'Man is a species' has simple supposition (*STL* I, 68). However, for Ockham, a term with simple supposition denotes a mental term, the act of thought, for example, the act of thinking of man in general. This of course is consonant with a certain sort of nominalism. But why then did he have to invoke the notion of merely confused supposition? The answer is, to paraphrase Ockham's argument, that if in 'Every man is an animal' 'an animal' had simple supposition, that is, supposited for an intention of the mind, then the sentence would say of every man that he is an intention of the mind. This is clearly absurd.[14]

To return to Ockham's anti-realist move: Ockham and others realised that whenever a general term is used it can be replaced (*salva veritate*) by an enumeration of its instances. For example, if William, Adam, Walter and John are all the men in the room, then

(1) Every logician is a man in the room

is at least materially equivalent to

2) Every logician is one of William, Adam, Walter and John.

Similarly, if the logicians are William, Walter, John and Richard, we can enumerate the subject term:

> Every (one of) William, Walter, John and Richard is a man in the room.

When this enumeration is complete—when we have descended completely to terms with discrete supposition—we can see whether the sentence is true or false (in this case false).

[14] *STL* I, 66: ll.26–41 and 137–42; see also Swiniarski's discussion, in "A New Presentation of Ockham's Theory of Supposition," ***Franciscan Studies***, 30 (1970), 205–6.

Ockham considered this equivalence between a term and an enumeration of its instances to be stronger than material. For Ockham's account of signification is extensional: the signification of a term consists of those things of which it can be truly predicated, that is, its extension.[15] Hence as a matter of meaning or signification, something is $\Phi$ just if it is this $\Phi$ or that $\Phi$ or..., that is, when it is one of the $\Phi$s. Ockham required that the equivalence of (1) and (2) be a consequence of a relation of synonymy: that a monadic predication such as 'William is a logician' actually mean 'William is William, Walter, John or Richard.'

Burleigh quite rightly pointed out that this view of sense is absurd (op. cit. p. 9). He said that on such a view no one could even move his finger without some sound thereby changing its meaning: "because by keeping a finger still the sound 'still' would signify the finger, while upon moving the finger that sound would not signify the finger. This is absurd." However, to reject this backing for the analysis of a term is not to reject the analysis. (1) and (2) are indeed materially equivalent. When this observation was added to the theory of distributive and determinate supposition inherited from the previous century, it was possible to dispense with simple supposition for the predicate of A-sentences. For some sentences it is possible to descend to a conjunction of sentences with each member of the enumeration of some general term denoted in some conjunct; for others it is possible to descend to a disjunction of such sentences. If each of these descents fails to give an equivalent, it is still possible to descend to a single sentence with the disjunctive enumeration of each member of the general term in place of that term, given by the term's signification. This is the second important aspect of Ockham's use of merely confused supposition. Celaya wrote: "Merely confused disjoint supposition [see § 6] is a term... for which it is permissible to descend and ascend *disiunctim*, and not otherwise... We say 'and not otherwise' to differentiate this mode from others. For certainly under any term it is permissible to descend and ascend *disiunctim*... So if we did not include that little phrase, 'and not otherwise,' we would find that any term would supposit merely confusedly; that is

[15] *STL* I, 33: p. 95. That Ockham's account of signification was intended as a theory of meaning is shown firstly by Burleigh's reaction (see below) and by such texts of Ockham's as *STL* I, 3: ll.20–1. where we read: "whatever is signified by an expression can be expressed equally by any synonym."

why we include it." And again later: "If one did not include that phrase, every term would have merely confused supposition."[16]

Thus the introduction of merely confused supposition and its corresponding descent *disiunctim* was important for two reasons: i) it allowed the truth conditions of all sentences to be given in a uniform manner, viz. as a Boolean combination of singulars; and ii) it provided a theoretical ground for Ockham's nominalism.

## 4. The Formalization

Our purpose in this section is to use modern quantification theory to give an insight into the workings of the theory of supposition. It has in fact, been denied that this is possible. For example, Matthews wrote: "One reason there can be no faithful rendering of suppositional descent in modern mathematical logic is this: since it is variables that are quantified in modern logic, any descent to singulars achieved by the elimination of quantifiers would have to be a descent to all x's, that is, to all the individuals within the universe of discourse... There could be no descent to say, men, and nothing else."[17] Our formalization will show this claim to be simply false. Matthews appears to be unaware of restricted quantification.

However, the main problems that those such as Boehner and Henry had in mind when they claimed that the mediaeval theory could not be formalized in modern logic, were different: they were i) the existential import of various Aristotelian forms, and ii) the fact that modern logic appears to have no equivalent to the notion of a disjunction of names.[18] The first of these problems we will return to in § 5 part 1. The latter we will discuss now.

Our central concern is with the notion of personal supposition. According to Ockham there are three categories of common personal supposition: determinate, confused and distributive, and merely confused. The first is analysed by a disjunctive descent, that is, by descent to a disjunction of singular sentences, the second by a con-

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[16] Celaya, op. cit., sign. b3vb and f1v.

[17] G. Matthews, "Ockham's Supposition Theory and Modern Logic," *Philosophical Review*, 73 (1964), 95–6.

[18] P. Boehner, *Medieval Logic* (Manchester, 1952), pp. 30–1; D. P. Henry, *Mediaeval Logic and Metaphysics* (London, 1973), III § 1.

junctive descent, that is, by descent to a conjunction of sentences. A case of merely confused supposition has no conjunction or disjunction of sentences as *analysans*. Ockham spoke of descent *ad disiunctum praedicatum*, later called descent *disiunctim*, to a disjunction of names. It was Boehner's contention that there is no equivalent notion in modern logic. However, we will show that there is. It can be represented by descent to a disjunction with narrow scope, that is, where a disjunction is necessarily substituted for a proper subformula of the original sentence. Not only does this resolve Boehner's problem, but we will see that it also explains a number of other facets of merely confused supposition: that with the other two modes it was thought to (and does) give a complete theory of supposition (at least in extensional contexts, and perhaps in all); that in the characterisation of merely confused supposition the conjunctive and disjunctive descents are usually explicitly excluded; and that Ockham and others failed to mention what would appear to be a natural addition to the trio of modes, a fourth mode of supposition given by a descent to a conjunction of names. We will deal with these matters in § 6.

Given a non-empty finite set of sentences $\{p_1, \ldots, p_n\}$ we can write '$\bigwedge_{1 \leq i \leq n} p_i$' for '$p_1 \wedge \ldots \wedge p_n$.' The truth conditions are then given naturally as:

$$\text{'}\bigwedge_{1 \leq i \leq n} p_i\text{' is true iff for all } 1 \leq i \leq n,\ p_i \text{ is true.}$$

Let $N = \{i\colon 1 \leq i \leq n\}$. Then we have

$$\text{'}\bigwedge_{i \in N} p_i\text{' is true iff for all } i \in N,\ p_i \text{ is true.}$$

But there is no reason why we sould restrict the set $N$ to be finite. Let I be any non-empty index set; then we can form the conjunction of members of any set of sentences indexed by I and give its truth conditions as follows:

$$\text{'}\bigwedge_{i \in I} p_i\text{' is true iff for all } i \in I,\ p_i \text{ is true.}$$

Arbitrary disjunction is defined dually and it is not surprising to find that all the standard properties of Boolean connectives, De Morgan equivalences, duality, and so on, hold for arbitrary conjunctions and disjunctions. Thus we can accomodate extensions of predicates of any cardinality.

Formally we give the definition of supposition for sentences in the language $L_{\infty\omega}$; '$\infty$' indicates that conjunctions and disjunctions may be of arbitrary size; '$\omega$' indicates that strings of quantifiers are finite. The customary first order language of everyday logical parlance is $L_{\omega\omega}$, the finitary fragment of $L_{\infty\omega}$.[19]

For ease of notation we often write $t_1(t_2)$ for $t_1 = t_2$ and $\Phi(A/B)$ for the formula obtained from $\Phi$ by substituting the formula B for a single distinguished occurrence of the formula A. Let $W$ be an arbitrary interpretation for $L_{\infty\omega}$, with domain W. To keep matters simple we will assume that every member of W has a name in $L_{\infty\omega}$. (This assumption is unnecessary if we are prepared to complicate matters by talking in terms of satisfaction rather than truth). And since no confusion should arise we will use the names of members of W as names of themselves (as the mediaevals did in the notion of material supposition).

Let $\psi(v)$ be a predicate with free variable v. The *extension* of $\psi(v)$ in $W$, $\psi_W$, is defined to be $\{w \in W : W \models \psi(w)\}$ (where $\psi(w)$ is the formula resulting from $\psi(v)$ by replacing all free occurrences of 'v' by 'w')[20] Hence for any $W$ such that $\psi_W$ is non-empty,

$$W \models \forall v\, (\psi(v) \leftrightarrow \bigvee_{w \in \psi_W} v = w) \tag{1}$$

The three modes of common personal supposition can now be defined in terms of descent, that is, in terms of truth-preservation when terms with discrete supposition (the names, w) are substituted for predicates. Let $\Phi$ be any sentence, $\psi(v)$ any predicate with free variable v where $\psi(t)$ is a subformula of $\Phi$, t any term, that is, variable or constant. Then the supposition of $\psi(t)$ in $\Phi$ is:

i) *determinate* iff for all $W$ such that $\psi_W$ is non-empty

$$W \models \Phi \leftrightarrow \bigvee_{w \in \psi_W} \Phi(\psi(t)/w(t)) \tag{D.}$$

ii) *confused* iff (D.) does not hold, and

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[19] Such infinitary languages are explained in J. Bell & A. Slomson, *Models & Ultraproducts* (Amsterdam, 1969), ch. 14.

[20] When $\psi(v)$ consists of a monadic predicate letter $\theta$ followed by a variable, we will write $\theta_W$ for $\psi_W$ for brevity.

a) *confused and distributive* iff for all $W$ such that $\psi_W$ is non-empty

$$W \models \Phi \leftrightarrow \bigwedge_{w \in \psi_W} \Phi(\psi(t)/w(t)) \qquad \text{(C.D.)}$$

and b) *merely confused* iff (C.D.) does not hold but for all $W$ such that $\psi_W$ is non-empty

$$W \models \Phi \leftrightarrow \Phi\,(\psi(t)/\bigvee_{w \in \psi_W} w(t)) \qquad \text{(M.C.)}$$

Since (1) holds, (M.C.) always holds (by the intersubstitutivity of material equivalent—our language is as yet purely extensional). Thus iib) gives a distinctive mode of supposition only by its exclusion of i) and iia). Note also that the descended form is materially equivalent to the undescended form, a form of analysis for which we argued in § 2. We will discuss the clause 'for all $W$ such that $\psi_W$ is non-empty' in § 5.

Note that for any $W$

$$W \models \forall x(x = w \rightarrow \theta(x)) \leftrightarrow \theta(w) \qquad (2)$$

and

$$W \models \exists x(x = w \wedge \theta(x)) \leftrightarrow \theta(w) \qquad (3)$$

With (1)–(3) and the equivalences specific to the modes of supposition we can always give the truth conditions of sentences containing terms with personal supposition using sentences containing only terms with discrete supposition. (However, in some cases this will require us to use the equivalence of $\forall x\Phi$ with $\forall x(x = x \rightarrow \Phi)$ and of $\exists x\Phi$ with $\exists x(x = x \wedge \Phi)$.)

Let us consider an example from *STL* I, 73: ll.30, 44–5:

The whole day some man was inside. (4)

This is ambiguous. Let $G$ be the actual world. (Henceforth, unless we say otherwise, all material equivalences are to be considered as true in $G$). Let 'Dx' read 'x is a time of day,' 'Hx' read 'x is a man,' and 'Ixy' read 'x is inside at y.' Then (4) can be represented either as A1) or as B1):

$$\forall y(Dy \rightarrow \exists x(Hx \wedge Ixy)) \qquad \text{(A1)}$$

$$\leftrightarrow \bigwedge_{w' \in D_G} \forall y(w'(y) \rightarrow \exists x(Hx \wedge Ixy)) \qquad \text{by (C.D.)}$$

$$\leftrightarrow \bigwedge_{w' \in D_G} \exists x(Hx \wedge Ixw') \qquad \text{by (2); call this (A2)}$$

$$\leftrightarrow \bigwedge_{w' \in D_G} \bigvee_{w \in H_G} \exists x(w(x) \wedge Ixw') \qquad \text{by (D.)}$$

$$\leftrightarrow \bigwedge_{w' \in D_G} \bigvee_{w \in H_G} Iww' \qquad \text{by (3).}$$

This exhibits firstly the fact that 'D' has distributive supposition in (A1), secondly the fact that 'H' has determinate supposition in each conjunct of (A2), and thirdly the full mechanics of descent to singulars.

Alternatively, we can descend firstly on 'H':

$$\forall y(Dy \rightarrow \exists x(Hx \wedge Ixy)) \qquad \text{(A3)}$$

$$\leftrightarrow \forall y(Dy \rightarrow \exists x(\bigvee_{w \in H_G} w(x) \wedge Ixy) \qquad \text{by (M.C.)}$$

$$\leftrightarrow \forall y(Dy \rightarrow \exists x \bigvee_{w \in H_G} (w(x) \wedge Ixy))$$

$$\leftrightarrow \forall y(Dy \rightarrow \bigvee_{w \in H_G} \exists x(w(x) \wedge Ixy))$$

$$\leftrightarrow \forall y(Dy \rightarrow \bigvee_{w \in H_G} Iwy) \qquad \text{(A4)}$$

$$\leftrightarrow \bigwedge_{w' \in D_G} \forall y(w'(y) \rightarrow \bigvee_{w \in H_G} Iwy)$$

$$\leftrightarrow \bigwedge_{w' \in D_G} \bigvee_{w \in H_G} Iww'$$

Naturally, we obtain the same fully descended form, as all the steps are equivalences. The descent indicates that 'H' has merely confused supposition in (A3), and that 'D' has distributive supposition in A4).

The other sense of (4) is given by:

$$\exists x(Hx \wedge \forall y(Dy \rightarrow Ixy)) \qquad \text{(B1)}$$

$$\leftrightarrow \bigvee_{w \in H_G} \exists x(w(x) \wedge \forall y\ (Dy \rightarrow Ixy))$$

$$\leftrightarrow \bigvee_{w \in H_G} \forall y(Dy \rightarrow Iwy) \qquad \text{(B2)}$$

$$\leftrightarrow \bigvee_{w \in H_G} \bigwedge_{w' \in D_G} \forall y(w'(y) \rightarrow Iwy)$$

$$\leftrightarrow \bigvee_{w \in H_G} \bigwedge_{w' \in D_G} Iww' \qquad \text{(B3)}$$

As expected, we find that 'H' has determinate supposition in (B1) and that 'D' has distributive supposition in each disjunct of (B2). The fully descended form (B3) differs from (A4) in just the way that the Fregean form (B1) differs from (A1). This is how the mediaevals revealed the ambiguity of (4).

Alternatively, we can start the descent under 'D' in (B1):

$$\exists x(Hx \wedge \forall y(Dy \rightarrow Ixy))$$

$$\leftrightarrow \exists x(Hx \wedge \forall y( \bigvee_{w' \in D_G} w'(y) \rightarrow Ixy))$$

$$\leftrightarrow \exists x(Hx \wedge \forall y \bigwedge_{w' \in D_G} (w'(y) \rightarrow Ixy))$$

$$\leftrightarrow \exists x(Hx \wedge \bigwedge_{w' \in D_G} \forall y(w'(y) \rightarrow Ixy))$$

$$\leftrightarrow \exists x(Hx \wedge \bigwedge_{w' \in D_G} Ixw')$$

$$\leftrightarrow \bigvee_{w \in H_G} \exists x(w(x) \wedge \bigwedge_{w' \in D_G} Ixw')$$

$$\leftrightarrow \bigvee_{w \in H_G} \bigwedge_{w' \in D_G} Iww'$$

So 'D' has merely confused supposition in (B1).

It cannot fail to be noted that the conjunction $\bigwedge_{w \in \psi_W}$ behaves just as does the restricted quantifier $(\forall x \psi)$ (Everything which $\psi$'s...), and dually for disjunction and the restricted existential quantifier $(\exists x \psi)$.

## 5. Comments on the Formalization

The above formalization was first presented in our paper, "The Formalization of Ockham's Theory of Supposition." There are a number of comments to be made on it.

1) There are four points in which the present account differs from the earlier one. We there allowed descent only under atomic predicates. We now allow descent under arbitrary predicates, that is, formulae with one free variable, formalizing complex terms. Ockham preceded us in this; indeed, according to him only the whole subject and predicate of a sentence can properly be said to have supposition, and not their parts. For example, he says of 'Every white man is white' that "neither [the first occurrence of 'white' nor 'man'] alone has what is properly called supposition. It is the whole composed of the two that supposits" (*STL* I, 73: 11.62–3). Our new formalization allows us to descend under 'white man' ('thing that is white and is a man'). To the extent that our formalization also gives supposition to 'white' and 'man' it is somewhat too liberal. This is not serious and Ockham's restriction could be accommodated; however, more importantly, Ockham's practice is to allow a broader sense of 'supposit' in which parts of the subject and predicate can be said to supposit. Our formalization now fits this broader sense precisely.

Secondly, in our present account we have introduced the universal quantification over interpretations $W$ in the criteria (D.), (C.D.), and (M.C.). In the earlier account the mode of supposition depended on contingent features of the actual world, $G$. This is wrong, since supposition is a matter of logic and should not depend on contingencies. Ockham, or his compilator, put it this way: "a variation or mutation of existing things [that is, a change in the interpretation $W$] does not cause a variation in the kind of supposition which is held by a term" (*EL*, p. 211). Our new definition captures this insight. However it should be emphasised that though the mode of supposition is world–independent, the actual *descensus* in each case depends on the actual world.

Thirdly, we have made the modes of supposition cumulatively exclusive. In fact this follows exactly the definitions given by Ockham in *EL*, pp. 209–10. The anonymous author of *Compilatio ex Buridano, Dorp, Ockam, Nicolai et alii nominalibus* (Paris, 1510), wrote: "These three descents, sc. conjunctive, disjunctive, and disjoint, belong in a certain order; because under any term allowing conjunctive descent one can descend disjunctively and disjunctly as well, and not conversely, and under any term allowing disjunctive descent one can also descend disjunctly, and not conversely."[21]

Moreover, under our old definition it was possible for the mode of supposition to be not uniquely defined. For if the extension of a term is a singleton then descent both to a degenerate conjunction and to a degenerate disjunction are possible. By the new account, and Ockham's explicit statement, a general term whose extension is a singleton in every interpretation has unambiguously determinate supposition. This answers Matthews' question as to the supposition of "monoreferential" terms.[22]

Fourthly, we have made the descent conditional upon the extension of a term's being non-empty. In our *Mind* paper we overlooked the fact that our definition made no clear sense if the extension of a term was empty. The mode of a term's supposition was supposed, we have said, to provide an account of the truth conditions of the sentence in which the term occurred. As so far formulated, it does not do this if the extension of the term is empty. It might be thought that it was a presupposition of mediaeval Aristotelian logic that any term mentioned had a non-empty extension. But this is false. It is a gross confusion to suppose that mediaeval logical theory was meant to *apply* only to non-empty terms.

One way of treating of the supposition of empty terms is to make sense of the notion of an empty disjunction or conjunction. This can be done in a natural way. According to the truth conditions given above an empty conjunction is (vacuously) true. Dually, an empty disjunction is false. With this understanding we can remove all clauses of the form 'such that $\psi_W$ is non-empty' from the definition of the modes of supposition. Descent is always possible, though sometimes it is to a vacuous and degenerate conjunction or disjunction.

This approach may seem a little artificial, and it seems implausible that any mediaeval logician thought of empty disjunctions and conjunctions. We think that Ockham would have appreciated such a notion. Moreover, in terms of the unification of the theory achieved

[21] f. clxiii rb. See also *Interpretatio in Summulas Petri Hispani... Georgii Bruxellensis* (Lugduni, 1509), f. 92va (folio actually marked 'LXXXIIII').

[22] G. Matthews, '*Suppositio* and Quantification in Ockham,' *Noûs*, 7 (1973), 21–2.

it may seem worthwhile. However, this approach has its problems. Consider 'Every chimaera is an animal'; in obvious notation:

$$\forall x(Cx \rightarrow Ax) \tag{1}$$

Whether or not we allow empty conjunctions, 'C' in (1) has distributive supposition, since the mode of supposition does not depend on fact, and in some worlds '$C_w$' will not be empty. But if we are allowed to descend to empty conjuncts, the descended form of (1) is:

$$\bigwedge_{w \in C_G} Aw \tag{2}$$

and since $C_G$ is empty, (2) is true.

However, Ockham held (1) to be false. So (2) cannot give its correct truth conditions. Ockham held that the subjects of the A and E forms had distributive supposition (resulting in conjunctive descent) and those of the I and O forms had determinate supposition (resulting in disjunctive descent); but that the A and I forms were false when the subject terms were empty, the E and O forms true. The idea of empty conjunctions being true, empty disjunctions false therefore cuts right across the ascription of truthvalues.

One way out is to say that 'Every chimaera is an animal' is more correctly represented by:

$$\exists xCx \wedge \forall x(Cx \rightarrow Ax) \tag{3}$$

In the first conjunct of (3), 'C' has determinate supposition since (assuming the use of degenerate disjunctions), for all $W$,

$$W \models \exists xCx \leftrightarrow \bigvee_{w \in C_W} w(w).$$

'$\exists xCx$' is false if $C_W$ is empty, true otherwise. Hence the fully descended form of (3) is:

$$\bigvee_{w \in C_G} w(w) \wedge \bigwedge_{w \in C_G} \bigvee_{w' \in A_G} w(w')$$

This gives the right truthvalues. But it will not do. For it does not attribute to the subject of the A-sentence, for example, a unique mode of supposition—it requires it to have one mode in one conjunct of its analysis, and another in the other. Ockham and others gave it distributive supposition, *simpliciter*. Can we then give an adequate account consistent with this (and other) unique ascriptions of both mode of supposition and truthvalue?

The simplest solution is just to accept that an empty term in a sentence, whilst it may have a mode of supposition, does not supposit for anything. This means that there is no descent possible and *a fortiori* the truth conditions of the sentence cannot be given by its descended form. There is therefore a gap in the theory of truth conditions, but this can be filled in by an *ad hoc* stipulation. Ockham does it as follows: "In affirmative sentences a term is always meant to supposit for something, and so if it supposits for nothing the sentence is false. However, in negative sentences the term is meant either not to supposit for anything or to supposit for something of which the predicate is truly denied, and so such a negative sentence has two causes of truth" (*STL* I, 72: ll.120–4). This is the answer to Matthews' question concerning the supposition of empty terms (loc. cit.).

2) Since the modes of supposition have been defined in terms of descent, it is not at all clear what is to be made of the notion of mobility. For it is commonly said that a term has mobile supposition when one can descend on it and immobile when one cannot. How then could a mode of supposition ever be immobile? To understand the situation one needs to trace the history of the term 'mobile.'

The distinction between mobile and immobile confused supposition makes one of its earliest appearances in Peter of Spain's *Tractatus*. It there marked the only division of modes of confused supposition: mobile confused was that of a general term not exhibiting determinate supposition on which one could descend conjunctively and immobile confused that of one on which one could not (op. cit., p. 83). But it does not follow that mobile confused supposition was that later called distributive; for as we have argued, distributive supposition requires conjunctive descent to an equivalent, while mobility we see, covers any term allowing conjunctive descent. Certainly all terms with distributive supposition allow such descent. But so also do some terms with merely confused supposition—in particular, the predicates of O–sentences. (As we shall see, Campanella divided merely confused supposition into mobile and immobile—see § 6, note 30).

The original meaning of 'mobile' was that later captured as 'distributed.' Ockham and others' use of it to separate off cases of distributive supposition permitting straightforward descent disguises this. As we saw, when descent first appeared it applied only to the inference of any singular—and so was in effect conjunctive. Once descent was generalised, the need for a separate notion of mobility lessened. So Ockham and others are found simply to use it to distinguish these cases of distributive supposition for which (conjunctive) descent is straightforward from those where it is not. For example, one cannot, they said, descend under 'man' in 'Every man except Socrates runs', even though it has distributive supposition.

However, even here Ockham is being inconsistent. For on his account it is the whole extreme 'man except Socrates' that should properly be said to have supposition, not just 'man'. Hence one should descend (conjunctively) under its extension (see § 5 (1) above)—as one can.

Of mobility, Ockham offers this principle: "If a term stands mobilely without any negation, it later stands immobilely when a negation is added to it. Take 'Socrates is every man'; here the word 'man' stands mobilely; therefore in 'Socrates is not every man', 'man' stands immobilely" (*STL* I, 74: ll.39–43). This cannot be understood to be true with Ockham's standard use of 'mobile'; it can only be interpreted correctly when 'mobile' is taken in its original connotation of the permitting of conjunctive descent. For a few lines earlier Ockham correctly observed that 'man' in 'Socrates is not every man' has determinate supposition; and so it permits only disjunctive descent, that is, one cannot descend to any singular: the term is immobile.

The famous dictum that "whatever makes the immobile mobile also makes the mobile immobile" (ibid.) became an integral part of the notorious doctrine of distribution, where we find that a term distributed (mobile) in one sentence is undistributed (immobile) in its contradictory, and vice versa. The predicate of the O–sentence, for example, is distributed or mobile. But one should not infer from this that it has distributive supposition, for the conjunctive ascent is impermissible.

3) Sometimes in presentations of supposition theory a priority rule is proposed, that is, a rule whereby one must descend first on terms with, say, determinate, then on those with distributive, and

finally on those with merely confused supposition. This seems to us a pointless device.

One reason for suggesting a priority rule is the claim that by adopting one, the notion of merely confused supposition can be dispensed with. By following the rule, it is said, we should always arrive at a fully descended form via determinate and distributive supposition.[23] However, this is simply incorrect; there are sentences in which the only supposition is merely confused. Consider, for example, 'Only mammals are not egg-layers.' Both 'mammals' and 'egg-layers' have merely confused supposition. The sentence is logically equivalent to the A-form, 'Any non-egg-layer is a mammal' in which the subject, 'non-egg-layer,' has distributive supposition. But the predicate of the original sentence is 'egg-layer,' and so this term has merely confused supposition by the mobility rule above. (*Pace* Weidemann,[24] the predicate must be taken to be 'egg-layer' and not 'non-egg-layer'; such use of infinite predicates as he makes would allow one to eliminate any one of the modes 'determinate,' 'distributive,' and 'merely confused,' at least in extensional contexts.)

A set of priority rules can be found in the early Renaissance writer, Domingo de Soto. He lists four rules "which indicate a certain order between the terms regarding their resolution," that is, analysis.[25] The second rule reads: "Under a term with merely confused disjoint supposition caused by a universality, one may not descend disjunctively before the said universality is resolved." One might interpret de Soto, as Ashworth seems to, to mean here that we cannot descend from, for example, 'Every man is an animal' to 'Every man is this animal or this animal and so on'; he is requiring descent firstly on

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[23] According to Swiniarski, op. cit. (see note 15), p. 209, and E. J. Ashworth, "Priority of Analysis and Merely Confused Supposition," *Franciscan Studies*, 33 (1973), 38–41, this suggestion is to be found in Geach's *Reference and Generality*. Geach has denied this in correspondence. Something like it can be found in E. Moody, *Truth and Consequence in Medieval Logic* (Amsterdam, 1953), pp. 47–8.

[24] H. Weidemann, "William of Ockham on Particular Negative Propositions," *Mind*, 88 (1979), 270–75.

[25] Domingo de Soto, *Summulae* or *Introductiones Dialecticae* (Burgis, 1529), f. 25r. This is the sole reference given in Ashworth, op. cit. (see note 23); in *Language and Logic in the Post-Medieval Period* (Dordrecht, 1974), p. 213; and "Multiple Quantification and the use of Special Quantifiers in Early Sixteenth Century Logic," *Notre Dame Journal of Formal Logic*, 19 (1978), 600.

'man' to obtain 'This man is an animal and this man is an animal and so on,' and then descent on 'animal.'

And that would be very odd; for merely confused disjoint supposition[26] is defined precisely by the permissibility of descent to a disjunct predicate. However, de Soto does not mean to rule out such a descent: he says explicitly in the passage above that one may not descend disjunctively (*disiunctive*), not that one may not descend *disiunctim*. The difference is shown, and the interpretation vindicated, by his example: "For example, this does not follow: every man is an animal... therefore every man is this animal or every man is this animal, and so on, although the converse ascent would be valid." Of course one cannot descend disjunctively on 'animal'; that and the converse ascent would show it to have determinate supposition, and it in fact has merely confused supposition. His point seems to be that if one wishes to descend disjunctively, so that finally one arrives at singular sentences without using disjunct predicates in the descent, one can, provided one proceeds in a certain order. The order is this: "In any sentence ascent or descent must firstly be made under the term with determinate supposition, then under the distributed term, then disjunctively under the term which supposited confusedly because of the distribution, which now becomes determinate [that is, in supposition] whenever the distribution is resolved" (f. 25rb). After resolving, that is, descending on, the distribution the term which had merely confused supposition because of the distribution (as 'animal' has in 'Every man is an animal' because of the quantifier 'every') becomes a term with determinate supposition, and so can subsequently be analysed disjunctively, that is, by a disjunction of sentences. However, the notion of merely confused supposition cannot be eliminated in this way. The reason is as before; all the terms in some sentences have merely confused supposition.

The strongest evidence in favour of Corcoran and Swiniarski's claim that descent is not to an equivalent sentence (see above, § 2) is that Ockham and others attributed confused and distributive supposition to the predicate of O–sentences. Our thesis that the definition of the modes of supposition is to be given in terms of descent to equivalent sentences requires us to charge Ockham and his suc-

[26] The distinction between disjoint and conjoint merely confused supposition is discussed in § 6 below (see note 30).

cessors with error.[27] De Soto notes the problem: "On a term suppositing distributively along with some term with determinate supposition, one cannot ascend before ascending on the determinate term... E.g., on the predicate of this sentence, 'Some man is not this animal,' it is not permissible to make an ascent: some man is not this animal, and some man is not this animal, and so on, therefore some man is not an animal." In fact an O–sentence is equivalent to the result of descending on its predicate to a disjunct term (as are all sentences) and to no other result of descent, and so its predicate supposits merely confusedly. But if one attributes distributive supposition to its predicate, one can disguise the failure of the converse ascent with de Soto's first priority rule: "Under such a distribution one should rightly descend first on the determinate term" (f. 25ra). For 'Some A is not B' is equivalent to 'This A is not B or that A is not B and so on,' and in each disjunct here 'B' has distributive supposition. Thus the right truth conditions are given whilst maintaining that the predicate of an O–sentence has distributive supposition.

Perhaps then, the priority rule functioned as a face–saver; an *ad hoc* rule whose sole function was to cover up an entrenched doctrinal mistake. It cannot serve to show any mode of supposition redundant.

## 6. *Suppositio Copulatim*

We must now turn to the subject of whether the theory of supposition we have presented is complete, that is, whether there are modes of supposition other than those we have defined. In particular, considerations of symmetry suggest that there is a mode that has been omitted. If there are descents to disjunctions and conjunctions of sentences and to disjunctions of terms, why not to conjunctions of terms too?

In the preface to the second edition of *Reference and Generality* Geach implied that the "conjunctive mode of reference... required for the symmetry of the medieval theory" is to be found in Paul of Venice. Certainly later mediaeval and Renaissance logicians were aware of the possibility of a fourth mode of supposition. Johannes Eckius wrote: "Whether to recognise collective supposition [this

[27] See our earlier paper in *Mind*, § 3(a).

fourth mode] or fewer modes is very doubtful. The first and foremost logicians do not recognise it; for example, the most learned William of Ockham, the repository of every part of logic, Marsilius of Inghen, than whom none is more acute in logic, Buridan, Thomas Maulfelt, George of Brussels, and many others. Others affirm it; for example, the now well-known school of Vienna and Erfurt cleave to it with might and main in their writings. Thomas de Clivis was among the first asserting this mode of supposition."[28]

But Paul of Venice never in fact introduces a fourth mode of supposition. What he does mention is *descensus copulatim*—descent to a conjunct term (*loc. cit.*, note 28). He gives as an example:

You are not every man (*Tu non es omnis homo*) (1)

and claims that one cannot infer *de copulato extremo*

You are not this man, or are you this man,...,

while one can infer a categorical sentence

You are not this man and this man...,

which is a correct provided we interpret the ambiguous sentence (1) as 'You are not all men,' and not as 'You are no man.'

[28] Johannes Eckius, *In Summulas Petri Hispani Extemporaria et Succincta* (Augustae Vindelicorum, 1516), f. XCIIIrb. This is the only reference given in Ashworth, *Language and Logic...* (see note 25), p. 212. Professor Ashworth has since kindly pointed out for us most of the following references. We are grateful to libraries in Aberdeen, Cambridge and Madrid for the opportunity to consult their copies. Those in favour of a fourth mode: Eckius, loc. cit.; de Soto, op. cit. (see note 25), ff. 20v, 25r; Jean de Celaya, op. cit. (see note 16), sign. b3vb; Campanella, *Dialecticorum* (in *Philosophia Rationalis Partes Quinque*, Paris, 1638), p. 355; Petrus Tartaretus, *Expositio in Summulas Petri Hispani* (Lugduni, 1501), sign. k3rb; Antonio Ramirez de Villascusa, *Abbreviationes Omnium Parvorum Logicalium* (Paris, 1510–13; in Aberdeen University Library), sign. b3ra; Phillippe du Trieu, *Manuductio ad Logicam* (London, 1662/1820), p. 116. Those arguing specifically against a fourth mode: Marsilius of Inghen, *Commentum in Primum et Quartum Tractatum Petri Hispani* (Hagenau, 1495), sign. q5v–q6v; George of Brussels, op. cit. (see note 21), f. 96va (folio marked 'LXXXVIII'); Johannes Dorp, *Joannis Buridani Perutile Compendium Totius Logicae* (Venice, 1499), sign. h5v–h6r; *Compilatio...* (see note 21), f. 157vb.

A similar example is to be found in Dorp's commentary on Buridan; Dorp notes that one cannot descend *disiunctim* on 'man' in

No animal is every man (*Nullum animal omnis homo est*) (2)

but that one can descend *copulatim*, to 'Every animal this man and this man and so for singulars is not' (*Omne animal iste homo et iste homo et sic de singulis non est*).[29]

A third plausible case of *descensus copulatim* is the occurrence of 'B' in

Some A loves every B (3)

Again, this is ambiguous; on one reading (that in which it is the same A doing the loving in every case) one cannot descend equivalently to

Some A loves this B and some A loves that B...

but only to

Some A loves this B and that B and...

(We should point out again that strictly speaking, for Ockham, the term 'B' does not have supposition since it is only part of the extreme. See § 5 part I. But a similar example where it does is 'Some A is every B.')

What are we to say of these examples? One possibility is to follow Paul of Venice and Dorp and define merely confused supposition as that mode of supposition whereby one can descend neither to a conjunction nor to a disjunction of sentences, but either to a conjunction or to a disjunction of terms. Recall that Ockham's criterion for merely confused supposition in *STL* II, 17–19 was simply that descent to a conjunction or disjunction of sentences was not permissible. Paul of Venice wrote: "Merely confused supposition is that of a general term in a sentence... from which one can descend neither with a conjunction nor with a disjunction of sentences, but *copulatim vel disiunctim*" (op. cit., p. 90).

[29] Dorp, loc. cit. (see note 28). The *Compilatio* repeats the passage. The same position is taken by Marsilius of Inghen and George of Brussels.

Alternatively, one can determine to distinguish a fourth mode of supposition corresponding to descent *copulatim.* This is what Eckius and some other later scholastics did.[30] Dorp mentioned both these possibilities: "And over and above these three modes [sc. determinate, distributive and merely confused] a fourth mode is suggested according to which there is a descent to a sentence with a conjunct term. The supposition of such term can be called collective supposition; or it can be called merely confused supposition—albeit merely confused supposition in a wide sense' (op. cit., h6ra).

However, there is a third and much simpler possibility, which is indeed suggested by our formalization. We noted in § 4 that the equivalence (M.C.) always holds. This proves that it is always possible to descend to a disjunct term, though to do this in natural language may require rephrasing the sentence into a logically equivalent one first. But Ockham was not averse to this and indeed held the mode of supposition to be invariant under logical equivalence.[31]

By way of illustration, consider (2) above. This can be written, with obvious notation, as:

$$\sim \exists x(Ax \wedge \forall y(My \rightarrow x = y))$$

By (M.C.) this is equivalent to

$$\sim \exists x(Ax \wedge \forall y(\bigvee_{w \in M_G} w(y) \rightarrow x = y))$$

$$\leftrightarrow \sim \exists x(Ax \wedge \forall y \bigwedge_{w \in M_G} (w(y) \rightarrow x = y))$$

$$\leftrightarrow \sim \exists x(Ax \wedge \bigwedge_{w \in M_G} \forall y(w(y) \rightarrow x = y))$$

$$\leftrightarrow \sim \exists x(Ax \wedge \bigwedge_{w \in M_G} w(x))$$

---

[30] The fourth mode was variously called *suppositio confusa tantum copulatim* (or *copulatum*), by Celaya, de Soto, Tartaret, and Villascusa; *suppositio collectivum* (or *collectiva*), by the *Compilatio*, Dorp, Eckius, and du Trieu; and *suppositio confusa tantum immobilis*, by Campanella.

[31] See for example, *STL*, I, 72: ll.207–10. In particular, note that whenever an author exhibits descent under 'A' in 'No A is B,' he descends under 'All A is not B'; the justification being that the mode of supposition of 'A' in the first sentence is the same as that of 'A' in the second, which is equivalent to the first provided that 'A' is non-empty, that is, if descent is possible at all.

Thus a conjunct descent can be achieved via the more fundamental disjunct descent: the last line is a formalization of 'No animal is this man and this man and...' (1) is treated similarly.

To perform the same analysis in natural language, we have to rephrase (2) as 'Every animal is not some man' (*Omne animal homo non est*) where in the Latin *homo* has wider scope than *non;* then one can descend *disiunctim* to 'Every animal this man or this man... is not' (*Omne animal iste homo vel iste homo... non est*).[32] We can treat (3) similarly. Writing it, with obvious notation, as

$$\exists y(Ay \wedge \forall x(Ex \rightarrow Byx))$$

we can descend by (M.C.) to

$$\exists y(Ay \wedge \forall x(\bigvee_{w \in B_G} w(x) \rightarrow Lyx)$$

and as before, this is equivalent to

$$\exists y(Ay \wedge \bigwedge_{w \in B_G} Lyw)$$

that is, Some A loves this B and this B and... To descend *disiunctim* in natural language, we rephrase (3), 'Some A loves every B,' as 'It is not the case that no A loves every B,' which as in the case of (2), is

It is not the case that every A does not love some B,

and descend to one reading of

It is not the case that every A does not love this B or...,

a descent *disiunctim*, where the disjunction has wider scope than 'does not love,' and so by De Morgan is equivalent to a conjoint descent, yielding 'It is not the case that every A does not love this B and that B and...,' that is,

[32] Dorp, op. cit., sign. h6rb: "It should be noted that before descent is made under '*homo*' in this sentence, '*nullum animal omnis homo est*,' that sentence must be analysed as '*omne animal homo non est*.' And then the descent must be made like this: *nullum animal omnis homo est*, therefore *omne animal iste homo vel iste homo non est*, which is true, as was the original."

Some A loves this B and that B and...

The fact that (M.C.) always holds shows that it is always possible to descend to a disjunct term. Sometimes in the presence of negation the underlying disjunction is turned into a conjunction in the surface form. However, with a little rearrangement it is always possible to express the descent *disiunctim*. Thus for extensional contexts at least, the theory of supposition we have presented is complete: every term that has neither determinate nor distributive supposition has merely confused supposition. What the early mediaevals who introduced the notion of merely confused supposition allowing descent to a disjunct predicate saw was that any predicate is extensionally equivalent to a list of its instances. Every sentence allows descent whereby a disjunction of names replaces a predicate; but sometimes we need to perform the descent on a logical equivalent.

## 7. Supposition in Intensional Contexts

We stressed in § 6 that the equivalence (M.C.) always holds. Thus the theory presented is complete in the sense that every predicate in every context must have one of the three modes of supposition. At least, this is true for extensional contexts. What however, of non-extensional contexts?

An example of the supposition of terms within non-extensional contexts, which Ockham discusses at length (*STL* I, 72: ll.139–205) is that of 'horse' in

I promise you a horse (1)

Ockham observes that it is, strictly speaking, not 'horse' which has supposition, but 'one who promises you a horse.' (Writing (1) as 'I am one who promises you a horse.') However, he then says that 'horse' may charitably be considered to have supposition, in fact merely confused supposition. For although one cannot descend from (1) to

I promise you this horse or I promise you that horse or...

nor *a fortiori* to the corresponding conjunction, one can descend to

I promise you this horse or that horse or...

taking the disjunction over all horses. (In fact, it must be over all present and future horses—but this is a nicety not relevant to our present discussion.) How are we to regard this observation of Ockham's? The problem is difficult. For one thing, there is no fully satisfactory account of the logical form of, or semantics for, intensional contexts. However, suppose we write (1) as "I promise there will be a horse which I give to you," and formalize this, with obvious notation, as

$$\mathrm{Pr}(\exists x(Hx \wedge Gx)) \tag{2}$$

Then Ockham's claim is that we can descend from this to

$$\mathrm{Pr}(\bigvee_{w \in H_G} Gw)$$

This seems *prima facie* incorrect. For despite the fact that for any $W$,

$$W \models \exists x(Hx \wedge Gx) \leftrightarrow \bigvee_{w \in H_G} Gw$$

we do not have

$$W \models \mathrm{Pr}(\exists x(Hx \wedge Gx)) \leftrightarrow \mathrm{Pr}(\bigvee_{w \in H_G} Gw)$$

It is, of course, precisely this failure of substitutivity of extensional equivalents which marks intensional contexts.

It could be, of course, that Ockham is making a mistake. This idea obtains some support from the fact that in the same section of *STL* Ockham says that (1) is equivalent to 'You will have a horse as a gift from me.' In this 'horse' clearly has determinate supposition, Hence, since modes of supposition are identical in equivalent sentences (see note 31), 'horse' supposits determinately in (1). However, there may be a different reason for Ockham's position. It is at least arguable that synonymy is a sufficient condition for intersubstitutivity in intensional contexts. So Ockham's belief that 'Hx' is actually synonymous with '$\bigvee_{w \in H_G} w(x)$' (where $G$ is the actual world), which we noted in § 3, would justify his claim that a merely confused descent is possible. Indeed, that Ockham considered the connection of (2) and (3) to be one of synonymy would explain why he apparently thought their equivalence not to need argument. He simply asserts that (3) follows from (2), and explains this by the fact that "terms following [terms such as 'promise'] have merely confused supposition in virtue of those verbs, and so it is not possible to descend disjunctively to singulars, but only by a disjunct predicate... The inference 'I promise you a horse, therefore I promise you this horse or that or that, and so on for all singulars, both present and future, is valid" (*STL* I, 72: ll.243–6,148–51).

## 9. Conclusion

We have now done all we set out to do. It may help to conclude by summarising our main points:

1) The theory of supposition is a theory of reference and the notion of descent by which the modes are characterised provides a way of giving the truth conditions of sentences in terms of truth functions of sentences whose terms have discrete supposition.

2) Ockham's notion of merely confused supposition, which allows descent *disiunctim* completes the theory of supposition which he inherited, in the sense that it shows that 1) can always be achieved in extensional contexts.

3) Ockham's theory extends to intensional contexts. However, its use in those contexts is problematic.

We also hope that the paper shows that techniques and notions drawn from modern logical theory can play an important role in helping to understand and evaluate mediaeval logic. We also think that the reverse is true. But that is a whole new subject.

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